St Stephen's School – Carramar Campus
Year 11 Mathematics Specialist **Test 3a**
Total Marks: 50 **Time Allowed: 60 mins** **Calculator Assumed**

Name: Answers **Mark:**

Teacher:________________________

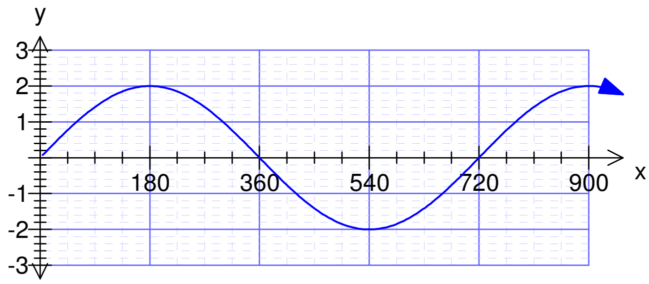
Parent Signature:

INSTRUCTIONS
Permitted equipment:

- Two calculators complying with Curriculum Council requirements
- Two A4 pages (both sides) of notes
- Stationery and drawing equipment
- Refer to the attached formula sheet

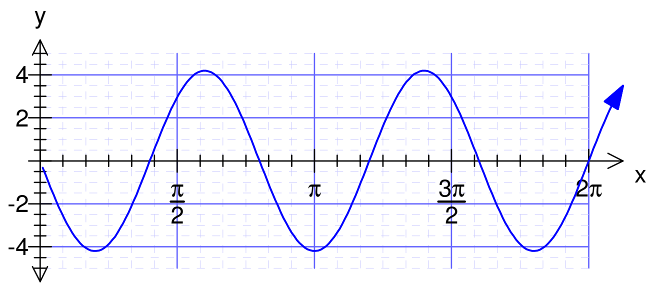
Question 1. **[2, 2, 2, 2, 2=10 marks]**

a. This graph is of the form $y = a\sin bx°$. State the value of a and b.



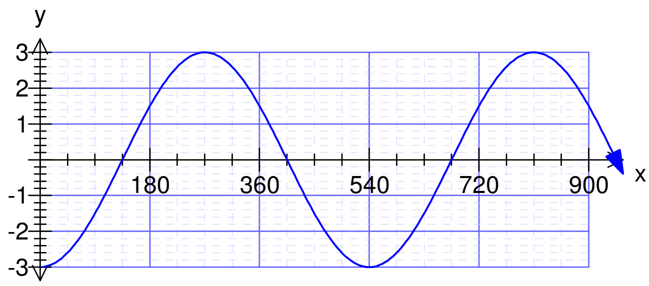
$a = 2$ ✓
$b = 0.5$ ✓

b. This graph is of the form $y = a\sin bx$. State the value of a and b.



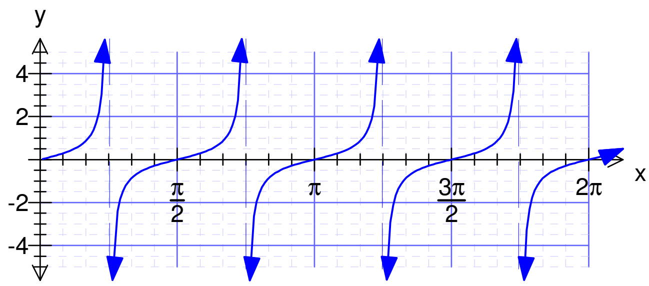
$a = -4.2$ ✓
$b = 2.5$ ✓

c. This graph is of the form $y = a\cos bx°$. State the value of a and b.



$a = -3$
$b = \frac{2}{3}$

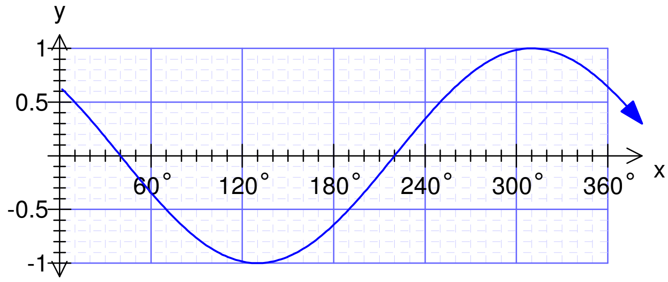
d. This graph is of the form $y = a\tan bx$. State the value of a and b.



$a = \frac{1}{2}$

$b = 2$

e. This graph is of the form $y = \cos(x - b)°$. State the two possible values of b nearest to zero (one positive and one negative).



$b = 310$

or

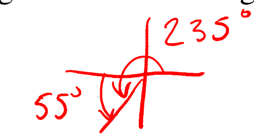
$b = -50$

Question 2. **[1, 1, 1, 1, 1, 1=6 marks]**

Express each of the following in terms of the sine, cosine or tangent of an acute angle:

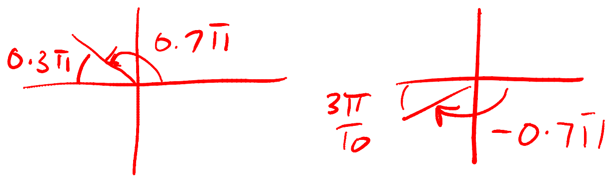
a. $\sin 235°$

$= -\sin 55°$ ✓



b. $\sin\left(-\frac{27\pi}{10}\right)$

$= -\sin \frac{3\pi}{10}$ ✓



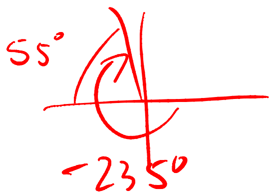
c. $\cos 235°$

$= -\cos 55°$ ✓

d. $\cos\frac{27\pi}{10}$

$= -\cos\frac{3\pi}{10}$ ✓

e. $\tan -235°$ $= -\tan 55°$ ✓



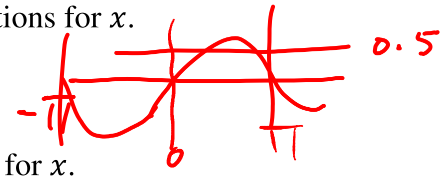
f. $\tan\frac{27\pi}{10}$ $= -\tan\frac{3\pi}{10}$ ✓

Question 3. **[1, 1, 1=3 marks]**

Consider the solutions to the equation $\sin x = a$ in the domain $-b \le x \le b$, (x and b in radians).

a. If $a = 0.5$, give any value of b that results in exactly 2 solutions for x.

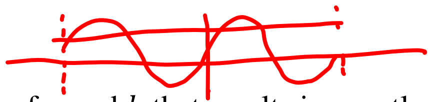
Any $b \in \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right)$



b. Give any value of a and b that results in exactly 4 solutions for x.

Any a in $(-1, 1)$ / $a = \pm 1$

$b = 2\pi$ / $b = 4\pi$



c. Given that b is a whole number multiple of π, give a value of a and b that results in exactly 5 solutions for x.

$a = \pm 1$

$b = 5\pi$

Question 4. **[2, 3=5 marks]**

a. How many solutions are there to $\tan 10x = -2.0$ in the domain $0 \le x \le 3600°$?

$\tan 10x$ has period $\frac{180}{10} = 18°$

$\frac{3600}{18} = \underline{\underline{200}}$

b. Give the least and greatest of these solutions.

$\tan^{-1} -2.0 = -63°$

$10x = -63°$

$x = -6.3°$

Not in domain...

Least $x = -6.3 + 18$
$= 11.7°$

Greatest $x = -6.3 + 3600$
$= 3593.7°$

Question 5. **[4 marks]**

Prove the identity

$$\frac{1-\sin^2 x+\sin^2 x\cos^2 x-\cos^2 x}{\sin x\cos x}=\sin x\cos x$$

LHS $\dfrac{\cos^2 x+\sin^2 x\cos^2 x-\cos^2 x}{\sin x\cos x}$ ✓

$= \dfrac{\sin^2 x\cos^2 x}{\sin x\cos x}$ ✓

$= \sin x\cos x$ ✓

$=$ RHS $\square$

Properly structured and set out ✓

Question 6. **[3, 3, 4=10 marks]** 2nd quadrant : cos & tan < 0

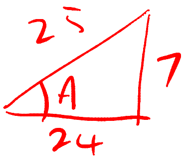
A and B are obtuse angles where $\sin A=\frac{7}{25}$ and $\sin B=\frac{12}{13}$. Find the following as exact values:

a. $\sin(A+B)$

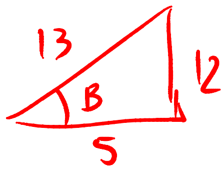
$= \sin A\cos B+\cos A\sin B$

✓✓ $= \frac{7}{25}\times\left(-\frac{5}{13}\right)+\left(-\frac{24}{25}\right)\times\frac{12}{13}$

✓ $= -\frac{323}{325}$



$\cos A = -\frac{24}{25}$



$\cos B = -\frac{5}{13}$

b. $\tan(A-B)$

$\tan A = -\frac{7}{24}$ $\tan B = -\frac{12}{5}$

$= \dfrac{\tan A-\tan B}{1-\tan A\tan B}$

✓✓ $= \dfrac{\left(-\frac{7}{24}\right)-\left(-\frac{12}{5}\right)}{1-\left(-\frac{7}{24}\right)\left(-\frac{12}{5}\right)}$

✓ $= \frac{253}{36}$

c. $\cos(2A + B) = \cos 2A \cos B - \sin 2A \sin B$ ✓

$= (2\cos^2 A - 1)\cos B - 2\sin A \cos A \sin B$ ✓

$= \left(2\left(-\frac{24}{25}\right)^2 - 1\right) \times \left(-\frac{5}{13}\right) - 2 \times \frac{7}{25} \times \left(-\frac{24}{25}\right) \times \frac{12}{13}$ ✓

$= \frac{1397}{8125}$ ✓

Question 7. **[4 marks]**

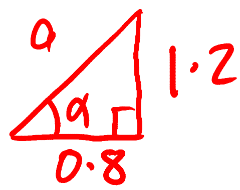
Express $1.2 \sin\theta + 0.8 \cos\theta$ in the form $a\cos(\theta - \alpha)$ for α an acute angle.

$a\cos(\theta - \alpha) = a\cos\theta\cos\alpha + a\sin\theta\sin\alpha$

$= a\sin\alpha \sin\theta + a\cos\alpha\cos\theta$

$a\sin\alpha = 1.2$ ✓

$a\cos\alpha = 0.8$



$a = \sqrt{1.2^2 + 0.8^2}$

$= 1.4$ (1 d.p.) (or $1.44222\ldots$) ✓

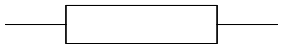
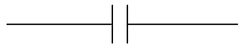
$\tan\alpha = \frac{1.2}{0.8}$

$\alpha = 56°$ $56.3099°\ldots$ or $0.98279^R\ldots$ ✓

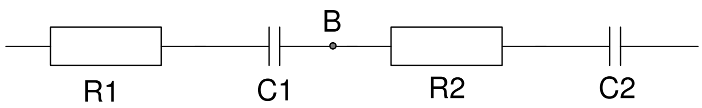
$1.2\sin\theta + 0.8\cos\theta = \underline{1.4\cos(\theta - 56°)}$ ✓

Question 8. [1, 2, 4=7 marks]

Alternating current circuits can be modelled using complex numbers for impedances. Resistors have positive real impedance while capacitors have positive imaginary impedance as shown in the table:

Type of Component	Symbol	Example Impedance z in ohms (Ω)
Resistor		$z = 10\Omega$
Capacitor		$z = 10\mathrm{i}\Omega$

a. A circuit has two resistors and two capacitors in series, as shown:



The four circuit elements have impedances $z_{\mathrm{R1}} = 150\Omega$, $z_{\mathrm{C1}} = 890\mathrm{i}\Omega$, $z_{\mathrm{R2}} = 220\Omega$, $z_{\mathrm{C2}} = 180\mathrm{i}\Omega$.

Calculate the total impedance by adding the impedance of the four elements.

$$Z = 150 + 890i + 220 + 180i$$
$$= (370 + 1070i)\Omega$$

b. In the circuit, the voltage at B can also be represented by a complex number. In this particular circuit, this voltage is given by

$$v_{\mathrm{B}} = 6.5\frac{z_{\mathrm{R2}} + z_{\mathrm{C2}}}{z_{\mathrm{R1}} + z_{\mathrm{C1}} + z_{\mathrm{R2}} + z_{\mathrm{C2}}}$$

Calculate v_{B}

$$V_B = 6.5 \times \frac{220 + 180i}{370 + 1070i}$$
$$= 1.389 - 0.856i$$

c. The power (in watts) consumed by a different circuit is given by

$$p = Re\left(\frac{v^2}{(47 + 18\mathrm{i})\Omega}\right)$$

Given that v (in volts) is a positive real number, show that the solution to $p = 1.0\mathrm{W}$ is $v = 7.3\mathrm{V}$.

$$\frac{V^2}{47 + 18i} = \frac{V^2}{47 + 18i} \times \frac{47 - 18i}{47 - 18i}$$
$$= \frac{V^2(47 - 18i)}{47^2 + 18^2}$$

$$= \frac{47}{2533} v^2 - \frac{18}{2533} i v^2 \quad \checkmark$$

$$Re\left(\frac{v^2}{47+18i}\right) = \frac{47}{2533} v^2$$

$$\frac{47}{2533} v^2 = 1 \quad \checkmark$$

$$v^2 = \frac{2533}{47}$$

$$v = \sqrt{\frac{2533}{47}} \quad \checkmark$$

$$= 7.3 \quad (\text{to } 1 \text{ d.p.})$$